**NTU SSS Economics HE1001**

**Problem Set 4: Game Theory and Oligopoly**

**This problem set will be discussed during the tutorials on 6-7 Nov**

**Game Theory**

1. Two major networks are competing for viewer ratings in the 8:00–9:00 pm and 9:00–10:00 pm slots on a given weeknight. Each has two shows to fill these time periods and is juggling its lineup. Each can choose to put its “bigger” show first or to place it second in the 9:00–10:00 pm slot. The combination of decisions leads to the following “ratings points” results:

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | Network 2 | |
|  |  | First | Second |
| Network 1 | First | 20, 30 | 18, 18 |
| Second | 15, 15 | 30, 10 |

(b) What will be the equilibrium if Network 1 makes its selection first? If Network 2 goes first?

Network 2 will play First regardless of what Network 1 chooses. Knowing this, Network 1 would play First if it could make its selection first, because 20 is greater than 15. So the equilibrium is (First, First).

1. In an ultimatum game, one player, the proposer, is endowed with $10. The proposer is tasked with splitting it with another player, the responder. Once the proposer communicates his decision, the responder may accept it or reject it. If the responder accepts, the money is split per the proposal; if the responder rejects, both players receive nothing. Both players know in advance the consequences of the responder accepting or rejecting the offer. Please predict the outcome.

It is an extensive form game; solving it backwards:

Step 1: in stage 2, the responder should accept any positive offer because if the responder rejects it, s/he would receive nothing.

Step 2: in stage 1, given the responder’s strategy in (1), the proposer should offer the smallest possible amount (e.g., 1 cent).

So, we predict that the proposer would offer the smallest possible amount and the responder would accept it.

**Oligopoly**

1. A monopolist can produce at a constant average (and marginal) cost of *AC*  *MC*  $5. It faces a market demand curve given by *Q*  53  *P*.

a. Calculate the profit-maximizing price and quantity for this monopolist. Also calculate its profits.

First solve for the inverse demand curve, *P*  53  *Q*. Then the marginal revenue curve has the same intercept and twice the slope:

*MR*  53  2*Q*.

Marginal cost is a constant $5. Setting *MR*  *MC*, find the optimal quantity:

53  2*Q*  5, or *Q*  24.

Substitute *Q*  24 into the demand function to find price:

*P*  53  24  $29.

Assuming fixed costs are zero, profits are equal to

**  *TR*  *TC*  (29)(24)  (5)(24)  $576.

b. Suppose a second firm enters the market. Let *Q*1 be the output of the first firm and *Q*2 be the output of the second. Market demand is now given by

***Q*1  *Q*2  53  *P*.**

**Assuming that this second firm has the same costs as the first, write the profits of each firm as functions of *Q*1 and *Q*2.**

When the second firm enters, price can be written as a function of the output of both firms:  
*P*  53  *Q*1  *Q*2. We may write the profit functions for the two firms:



and



c. Suppose (as in the Cournot model) that each firm chooses its profit-maximizing level of output on the assumption that its competitor’s output is fixed. Find each firm’s “reaction curve” (that is, the rule that gives its desired output in terms of its competitor’s output).

Under the Cournot assumption, each firm treats the output of the other firm as a constant in its maximization calculations. Therefore, Firm 1 chooses *Q*1 to maximize **1 in part bwith *Q*2 being treated as a constant. The change in **1 with respect to a change in *Q*1 is



This equation is the reaction function for Firm 1, which generates the profit- maximizing level of output, given the output of Firm 2. Because the problem is symmetric, the reaction function for Firm 2 is

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d. Calculate the Cournot equilibrium (that is, the values of *Q*1 and *Q*2 for which each firm is doing as well as it can given its competitor’s output). What are the resulting market price and profits of each firm?

Solve for the values of *Q*1 and *Q*2 that satisfy both reaction functions by substituting Firm 2’s reaction function into the function for Firm 1:



By symmetry, *Q*2  16.

To determine the price, substitute *Q*1 and *Q*2 into the demand equation:

*P*  53  16  16  $21.

Profit for Firm 1 is therefore

*1*  *PQ1*  *C*(*Q1*)  *i*  (21)(16)  (5)(16)  $256.

Firm 2’s profit is the same as Firm 1. *2*  $256.

1. Following Question 1, now we will use the Stackelberg model to analyze what will happen if one of the firms makes its output decision before the other. Suppose Firm 1 is the Stackelberg leader (that is, makes its output decisions before Firm 2). How much will each firm produce, and what will its profit be?

Firm 1, the Stackelberg leader, will choose its output, *Q*1, to maximize its profits, subject to the reaction function of Firm 2:

max **1  *PQ*1  *C*(*Q*1),

subject to



Substitute for *Q*2 in the demand function and, after solving for *P*, substitute for *P* in the profit function:



To determine the profit-maximizing quantity, we find the change in the profit function with respect to a change in *Q*1:



Set this expression equal to 0 to determine the profit-maximizing quantity:

53  2*Q*1  24  *Q*1  5  0 => *Q*1  24.

Substituting *Q*1  24 into Firm 2’s reaction function gives *Q*2:



Substitute *Q*1 and *Q*2 into the demand equation to find the price:

*P*  53  24  12  $17.

Profits for each firm are equal to total revenue minus total costs, or

**1  (17)(24)  (5)(24)  $288, and

**2  (17)(12)  (5)(12)  $144.